Mathematics 1 AESB1110-15: Test 2

October 21, 2016

Rules: You have 1 hour for this test. Write clearly and show steps/proofs.

Question 1: Find the first *three nonzero terms* of Maclaurin's expansion $\boxed{2\,p.}$
for the function $f(x) = \sin(x^2)$ and the appropriate Peano's remainder.
Hint: you may use standard Taylor expansions.

Question 2: Use Lagrange's remainder to estimate the range of x for $\boxed{2\,p.}$
which the *three-term* Maclaurin's approximation of the function
$f(x) = e^{-ax}$, where $a > 0$ is some constant and $x \geq 0$, gives the error smaller
than 0.001. *Hints:* (1) the final answer will depend on the parameter a, (2)
do not forget that $x \geq 0$.

Question 3: Find the derivative $f'(x)$ of the following function: $\boxed{2\,p.}$

$$f(x) = \int_{x^2}^{\cos^2(x)} \ln(t^2)\, dt$$

Question 4: Calculate the following integral: $\boxed{2\,p.}$

$$\int \sin^{-1}(x)\, dx$$

Hint: $\frac{d}{dx}\left[\sin^{-1}(x)\right] = \frac{1}{\sqrt{1-x^2}}.$

Question 5: Calculate the following improper integral (if it converges):
$\boxed{2\,p.}$

$$\int_1^9 \frac{1}{\sqrt[3]{x-1}}\, dx$$